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$$a^2c^2+2c(2x^3+3axy)-y^2(4ay+3x^2)=0.$$

This is the primitive equation. If the values of c are equal, then

$$(2x^3+3axy)^2+a^2y^2(4ay+3x^2)=0;\ i.\ e.,\ (x^2+ay)^3=0.$$

When the discriminating equation has a factor cubed, this factor equated to zero gives the cuspidal locus of the given family of curves. In fact, in the present case there is a cusp at the point ($a^{\frac{2}{3}}c^{\frac{1}{3}}$, $-$ $a^{\frac{1}{3}}c^{\frac{2}{3}}$).

GEOMETRY.

260. **Proposed by W. J. GREENSTREET, M. A., Stroud, England.**

Perpendiculars to the radius vector are drawn through points on $r=a+b\cos n\theta$. Find the radius of curvature of their envelope at a point at a given distance from the origin.

I. **Solution by G. B. M. ZERR, A. M., Ph. D.**

$r=a+b\cos n\theta=R\cos(\phi-\theta)=p$..........(1).

Differentiating (1) we get, $-bn\sin n\theta=R\sin(\phi-\theta)$..........(2).

$\therefore R^2=(a+b\cos n\theta)^2+b^2n^2\sin^2 n\theta$..........(3).

$\therefore R^2=p^2+b^2n^2-n^2(p-a)^2$..........(4).

(3) and (4) are both equations to the envelope.

Radius of curvature$=\rho=R(dR/dp)$.

$\therefore \rho=p+an^2-n^2p=r+an^2-n^2r=an^2+(1-n^2)(a+b\cos n\theta)$
$=a+b\cos n\theta-bn^2\cos n\theta=a-b(n^2-1)\cos n\theta$.

II. **Solution by G. W. GREENWOOD, M. A.**

Let $(p,\ \alpha)$ be the foot of the perpendicular from the origin upon the tangent at a point P of the envelope. Then at P,

$$\rho=p+d^2p/d\alpha^2=a+b\cos n\alpha-bn^2\cos n\alpha,$$

since $\quad p=a+b\cos n\alpha$.

MECHANICS.

182. **Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

I have a tank, the lower part of which is a hemisphere 22 feet in diameter. The rest is a cylinder 22 feet in diameter, and altitude 28 feet. This tank is connected with the earth by a vertical stand-pipe 10 inches in diameter, 130 feet long, extending 2 feet into the tank. The tank is filled by a 2½ inch pipe 65 feet long, having one right-angled elbow delivering the water into the bottom of the stand-pipe from a steam pump under 96 pounds gauge pressure. How long will it take to fill the pipe?

Solution by the PROPOSER.

Let h=effective head of water, W=weight of water discharged, $l=65$ feet =length of feed pipe, $d=2\frac{1}{2}$ inches$=\frac{5}{24}$ feet=diameter of feed pipe. Then hW=total energy consumed.

$\frac{Wv^2}{2g}$=kinetic energy giving motion to the water, $\frac{flv^2W}{2dg}$=energy of surface friction in pipe, $\frac{\beta v^2W}{2g}$=energy of resistance at entrance of pipe, $\frac{\delta v^2W}{2g}$=energy of resistance at elbow. $\therefore hW=\left(\frac{fl}{d}+\beta+\delta+1\right)\frac{Wv^2}{2g}$.

From many experiments, $f=.030268$, $\beta=.505$, $\delta=.9846$.

$\therefore h=(9.443616+.505+.9846+1)\frac{v^2}{2g}$; $\quad \frac{1}{2g}=.0155$.

$\therefore h=.184965v^2$ or $v=2.32517\sqrt{h}$.

$Q=\frac{\pi}{4}d^2v$=cubic feet delivered per second$=\frac{\pi\times\frac{25}{4}\times3600\times2.32517\sqrt{h}}{4\times144}$ $=285.342\sqrt{h}=C\sqrt{h}$ suppose=cubic feet per hour.

$C\sqrt{h}\,dt$=cubic feet delivered in time dt; πy^2dx=height dx of tank filled in time dt.

$\therefore C\sqrt{h}\,dt=\pi y^2dx$. $\quad\therefore t=\frac{\pi}{C}\int\frac{y^2dx}{\sqrt{h}}$.

Let $p=96$ pounds=gauge pressure of water on pump. Since one foot of water$=.43302$ pounds per square inch, p pounds$=2.30936p$ feet of water.

$\therefore h=2.30936p-130$. Let $B=2.30936p-128$; $B'=2.30936p-139$.

Let t=time of filling whole tank; t_1=time of filling the bottom two feet; t_2=time of filling the rest of the hemisphere; t_3=time of filling the cylinder.

The equation to the circle whose revolution about a diameter generates the sphere is $x^2+y^2=22x$.

$$\therefore t_1=\frac{\pi}{C\sqrt{h}}\int_0^2(22x-x^2)dx=\frac{124\pi}{3C\sqrt{h}};\quad t_2=\frac{\pi}{C}\int_2^{11}\frac{(22x-x^2)\,dx}{\sqrt{(B-x)}}.$$

$$\therefore t_2=\frac{\pi}{15C}\Big[(6x^2-220x+8Bx-440B+16B^2)\sqrt{(B-x)}\Big]_2^{11}$$

$$=\frac{\pi}{15C}\Big[(16B^2-352B-1694)\sqrt{(B-11)}-(16B^2-424B-416)\sqrt{(B-2]}\;\Big]$$

$$t_3=\frac{121\pi}{C}\int_0^{28}\frac{dx}{\sqrt{(B'-x)}}=\frac{242\pi}{C}[\sqrt{(B')}-\sqrt{(B'-28)}].$$

$$t=t_1+t_2+t_3=\frac{\pi}{C}\Big[\frac{124}{3\sqrt{h}}+\tfrac{1}{15}(16B^2-352B-1694)\sqrt{(B-11)}$$

$$-\tfrac{1}{15}(16B^2-424B-416)\sqrt{(B-2)}+242[\sqrt{(B')}-\sqrt{(B'-28)}]\Big].$$

$p=96$, $h=91.7=92$ say, $B=94$, $B'=83$.

$\therefore t=\pi/C\,(4.3093+90.3161+409.9964)=1.76848\pi=5$ hours, 33 minutes, 21 seconds.

The actual time observed for filling this tank to within 4 inches of the top was $5\frac{1}{2}$ hours.

MISCELLANEOUS.

149. Proposed by **F. P. MATZ, Ph. D., Sc. D.**

Given $\sin^{-1}u+\sin^{-1}\frac{1}{2}u=\frac{1}{4}\pi$, to find u.

Solution by **J. EDWARD SANDERS.**

By use of the addition theorem, we have

$$\tfrac{1}{2}\sqrt{2}=u.\tfrac{1}{2}\sqrt{(4-u^2)}+\tfrac{1}{2}u.\sqrt{(1-u^2)}.$$

Squaring twice and arranging, we get the trinomial $17u^4-20u^2=-4$, or $u=\pm\sqrt{(\frac{10}{17}\pm\frac{4}{17}\sqrt{2})}$. Whence $u=\pm.50544945$......... or $\pm.95968298$.........

The first value is the one solving the question.

Also solved by R. D. Carmichael, G. W. Greenwood, A. H. Holmes, L. E. Newcomb, J. Scheffer, W. L. Tryon, G. B. M. Zerr, and the Proposer.

150. Proposed by **T. N. HAUN, Mohawk, Tenn.**

If $\dfrac{\sin\phi}{\sin\psi}=m$, find maximum and minimum value of $\dfrac{\sin(\phi+\theta)}{\sin(\psi+\theta)}$, where θ is known.

I. Solution by **A. H. HOLMES.**

$$\frac{\sin\phi}{\sin\psi}=m. \quad \therefore \sin\phi=m\sin\psi \text{ and } \cos\phi=\sqrt{(1-m^2\sin^2\psi)}.$$

$$\therefore \frac{m\sin\psi\cos\theta+\sin\theta\sqrt{(1-m^2\sin^2\psi)}}{\sin\psi\cos\theta+\sin\theta\sqrt{(1-\sin^2\psi)}}=\text{maximum or minimum.}$$

Differentiating, etc.,

$$\sin^4\psi-\frac{2\cos^2\theta(m^2+\sin^2\theta+\cos^2\theta)}{(m^2-\sin^2\theta+\cos^2\theta)^2+4\sin^2\theta\cos^2\theta}$$

$$=-\frac{\cos^4\theta}{(m^2-\sin^2\theta+\cos^2\theta)^2+4\sin^2\theta\cos^2\theta}.$$

$$\therefore \sin\psi=\frac{\cos\theta}{\sqrt{(m^2-2m\sin\theta+1)}} \text{ for maximum,}$$

$$\text{and } \sin\psi=\frac{\cos\theta}{\sqrt{(m^2+2m\sin\theta+1)}} \text{ for minimum.}$$